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RADIOMETER FORCE AND DIMENSIONS OF APPARATUS

BY

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It is a well-known fact that the radiometer force is dependent on the pressure of the gas, the temperature differences, and the dimensions of the apparatus.

In the following we shall describe some measurements of the radiometer force on a narrow platinum band blackened on one side and placed in the axis of a cylinder. The object is to determine how the radiometer force varies with the diameter of the cylinder.

The work has been carried out in Professor MARTIN KNUDSEN's laboratory and forms a kind of continuation of Professor KNUDSEN's work on radiometer forces and the coefficient of accommodation published in 1930[1], his method of measuring the radiometer forces described in that work being applied without alteration.

A vertical section of the apparatus employed is shown in fig. 1.

PP is the platinum band which is soldered to the two copper wires K_1 and K_2. K_1 is insulated and passed airtightly through the thick brass plate *Bp*, by means of a glass tube *G*, tightened with Picein. K_2 is in conductive connection with *Bp* through the frame formed of the brass rods M_1, M_2, and M_3.

Around *PP* may be placed copper cylinders of various diameters. Fig. 1 shows one of the 4 cylinders (*C*) employed.

[1] D. K. D. V. S. math.-fys. Medd. XI, 1.

The cylinder stands on the bottom plate in a circular groove; the upper end is fastened to a movable cross-piece which can be fastened to the vertical brass rods by means of a couple of nuts.

The whole apparatus is enclosed in the glass vessel *Gl*, which is cemented to the bottom plate with Picein so as to be airtight.

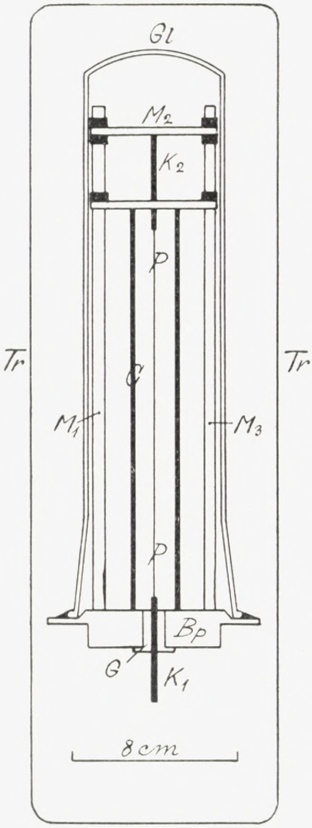

Fig. 1.

A glass tube connects this apparatus with the other parts of the system which are: a pipette system, which, in connection with a mercury manometer, serves to introduce a known amount of gas into the apparatus, and a pump aggregate.

In the glass tube there is a gas trap which is kept cool by means of liquid air during the measurements.

The temperature *t* of the copper cylinder is read on a mercury thermometer whose bulb by means of Woods metal is fused into a vertical hole in a small copper block soldered to the copper cylinder close to its lower end.

In order to enable the copper cylinders to be placed round the platinum band it was necessary to saw a strip c. 3 mm wide out of the cylinder wall; in addition a 10 mm hole was drilled through the middle of each cylinder. Through this hole the platinum band is observed through a reading-microscope furnished with an eyepiece-micrometer.

The 4 copper cylinders are all 20 cm long, the cylinder wall is c. 2.5 mm thick, and the internal diameters are respectively $d = 4.11$ cm, 3.21 cm, 2.24 cm, and 1.27 cm.

The platinum band was cut out of a larger foil of the ordinary trade article, it was blackened with platinum black on one side. The constants are: the length = 15.63 cm, the width $B = 0.2540$ cm, the thickness determined by weighing $\tau = 0.000299$ cm, the electrical resistance at 0° $R_0 =$ 2.4869 Ohms.

The temperature coefficient for the resistance was measured for a strip cut from the same plate. $\frac{R_t}{R_0} = 1 + \alpha t - \beta t^2$ where $\alpha = 0.003085$ $\beta = 65 \cdot 10^{-8}$.

In Professor KNUDSEN's paper it is described in detail how a radiometer force is measured by magnetic compensation[1].

In what follows

p denotes the pressure of the gas (hydrogen) in the apparatus measured in Bars.

i_1 Amp. denotes the heating current through the platinum band when the resistance is r Ohms and the mean temperature $T°$.

i_2 Amp. denotes the current which produces the compensating magnetic field in the coil Tr (Fig. 1).

V Volts denotes the drop in potential through the platinum band read on the compensation apparatus; hence $i_1 = \frac{V}{r}$.

$T_1 = T - t$ is the temperature difference between the platinum band and the copper cylinder.

K_1 Dyn/cm is the force exerted by the magnetic field on 1 cm of the length of the platinum band $= k \cdot i_1 i_2$, the numerical value of the constant k being determined

[1] L. c. p. 32.

at 0.2375 by measurement of the magnetic field of the coil.

R_1 Dyn/cm is the radiometer force per centimetre of the length of the band.

With these designations we have:

$$\frac{R_1}{T_1} = 0.2375 \left(\left(\frac{i_1 i_2}{T_1} \right)_p - \left(\frac{i_1 i_2}{T_1} \right)_0 \right)$$

when the indices p and 0 denote that the quantity in question has been measured respectively at the pressure p and in vacuum.

The measurements furnish data for the calculation of the amount of heat given off by conduction through the gas in the apparatus from the surface of the platinum band. The total amount of heat generated per sec. is $i_1^2.r$ Watts. As regards corrections for radiation and for conduction from the ends of the band the reader is referred to Professor KNUDSEN's paper.

Q being the corrected value of the heat conduction per sec. from the whole of the platinum band measured in Watts, we put

$$\frac{q}{T_1} = \frac{Q}{T_1} \cdot \frac{10^7}{3.97}$$

Erg/cm^2 sec. degree; hence q denotes the rate of heat loss by 1° temperature difference from 1 cm^2, the area of the band being = 3.97 cm^2 = the length multiplied by the width.

Results of the Measurements.

For each pressure p, R_1 and Q were measured for 4 different values of the temperature difference, each measurement being the mean value of two separate measurements.

The table below gives the values of $10^5 \frac{R_1}{T_1}$. Since $\frac{R_1}{T_1}$ only varies slightly with T_1, this quantity is only given over each column where the mean value has been rounded off to whole degrees, whereas the division $\frac{R_1}{T_1}$ has of course been made with the exact value of T_1.

Owing to exigencies of space the values for the loss of heat are not tabulated.

Table of $10^5 \frac{R_1}{T_1}$.

d = 4.11 cm					d = 3.21 cm				
p	48°	85°	122°	160°	p	48°	85°	122°	160°
5.76	53	41	37	35	5.26	48	41	34	32
11.49	92	77	70	68	10.50	81	74	67	62
17.19	132	111	103	99	15.71	117	104	95	90
22.3	166	144	133	127	20.9	146	131	119	113
28.5	186	171	159	152	26.1	172	158	143	137
34.2	214	194	182	174	31.2	194	180	163	154
21.1	157	140	127	120	36.4	211	199	182	173
41.1	262	228	209	200	20.5	152	136	112	108
60.1	312	281	259	248	40.0	240	222	196	183
103.6	370	334	310	298	70.0	334	306	273	257
192	363	331	314	307	91.6	367	344	328	317
276	327	292	282	281	98.3	373	345	309	293
419	258	231	228	232	153	379	363	329	316
453	238	222	217	223	156	380	360	351	342
564	201	181	185	192	282	324	316	295	288
759	137	135	142	150	305	322	308	307	307
1021		105	108	113	405	266	262	250	248
1374		76	76	80	446	258	248	252	257
1808		42	51	60	634	182	185	183	185
2053		43	33	34	910	134	132	143	150
3696		8	15	19	930	122	122	128	136
3771		36	23	19	1253	76	81	85	97
8230		—1	—4	0	1650	46	51	56	63
					3175			23	26
					4290			0	4
					6680			—3	—3

Table of $10^5 \frac{R_1}{T_1}$.

d = 2.24 cm					d = 1.27 cm				
p	48°	85°	122°	160°	p	48°	85°	122°	160°
6.06	57	48	44	38	6.22	50	53	50	45
12.09	91	85	78	72	12.41	107	99	94	86
18.09	137	126	115	109	18.57	159	144	133	124
24.1	175	159	143	133	24.7	206	189	172	165
30.0	206	191	172	161	30.8	250	228	212	202
36.0	238	217	197	190	36.9	288	266	246	236
41.9	267	247	222	209	26.8	228	214	197	187
21.6	158	146	126	116	53.4	388	361	335	322
42.1	263	248	220	205	77.5	497	462	430	416
56.3	315	296	267	260	100.3	567	532	498	483
61.5	338	317	281	262	124.0	612	580	546	542
85.8	413	386	353	336	161	638	611	590	573
107.8	434	417	386	367	242	642	641	603	587
125.0	460	433	398	381	354	582	587	575	568
168	433	418	388	374	460	512	523	520	518
210	444	438	412	398	568	465	470	481	493
308	391	394	378	373	700	385	396	403	415
400	343	347	335	336	929	300	315	328	341
614	247	256	255	257	1147	242	256	269	286
819	180	190	196	202	1744	141	146	158	170
1443	91	101	109	117	2520	107	96	106	121
2153	48	53	58	59	3908	43	29	49	56
4740	3	2	7	9	5365	23	17	25	30
4792	8	0	5	12					

If $\log p$ is plotted against $\frac{R_1}{T_1}$ in a rectangular system of coordinates, we shall obtain, in all, 16 radiometer curves, viz. 4 for each cylinder, corresponding to the 4 values of T_1.

Fig. 2 shows the 4 radiometer curves obtained for $T_1 = 122°$ for the 4 cylinders d = 4.11 cm, 3.21 cm, 2.24 cm, and 1.27 cm. The same figure shows 4 curves representing the course of the heat conduction for the 4 cylinders and for the same value of T_1. The curves have all been drawn as smoothly as possible through the points representing the separate measurements. These points have been omitted so as not to crowd the figure, but in fig. 3 a single radiometer curve and a curve of the heat conduction have

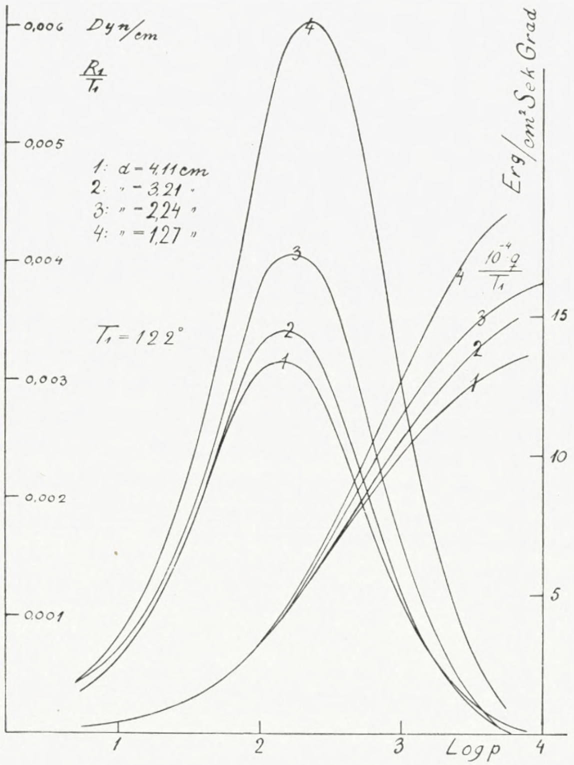

Fig. 2.

been drawn with the points of measurement included. This figure will convey an impression of the mutual agreement between the measurements.

From fig. 2 it will appear that the radiometer force increases rapidly with a decrease in the diameter of the surrounding cylinder, and this applies especially to high pressures and pressures about the maximum. At low pressures the effect of the variations in the diameter of the cylinder is much less, without, however, entirely vanishing.

By means of the radiometer curves the pressure p_m can be found at which R_1 has its maximum R_{1m}, if for each

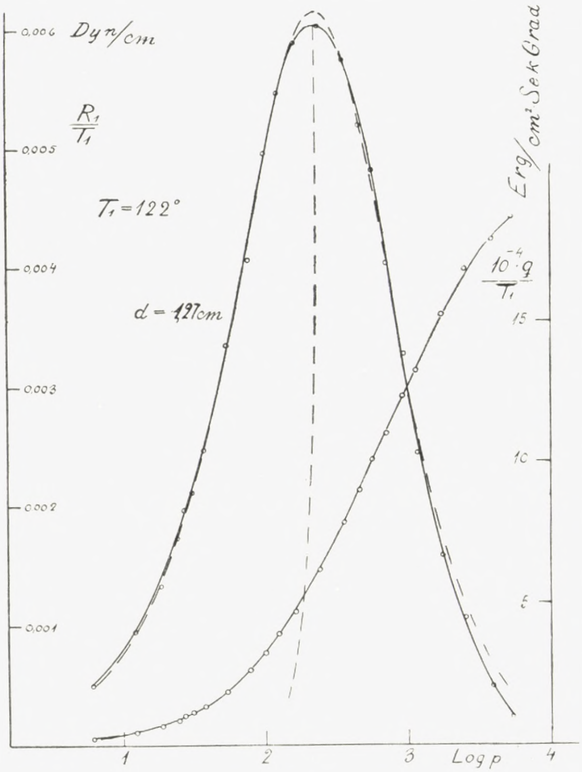

Fig. 3.

radiometer curve the line be drawn (the middle line) which connects the mid-points of the horizontal distances between the ascending and the descending part of the curve and its point of intersection with the curve be determined, the coordinates to the point of intersection being $\log p_m$ and $\frac{R_{1m}}{T_1}$.

In fig. 3 the middle line is dotted.

The results of this graphic determination of p_m and $\frac{R_{1m}}{T_1}$ are given in the following table.

d	$T_1 =$	48°	85°	122°	160°
4.11 cm..	$p_m =$	129	135	138	148
	$\Delta =$	+ 1	+ 1	— 3	0
	$10^5 \cdot \frac{R_{1m}}{T_1} =$	375	342	321	309
	$\Delta =$	+ 11	— 2	— 2	+ 6
3.21 cm..	$p_m =$	135	144	155	159
	$\Delta =$	0	+ 1	+ 4	+ 1
	$10^5 \cdot \frac{R_{1m}}{T_1} =$	382	361	342	328
	$\Delta =$	— 14	— 16	— 14	— 7
2.24 cm..	$p_m =$	151	166	170	182
	$\Delta =$	0	+ 5	0	+ 3
	$10^5 \cdot \frac{R_{1m}}{T_1} =$	452	432	408	394
	$\Delta =$	— 10	— 10	— 13	— 7
1.27 cm..	$p_m =$	195	214	229	234
	$\Delta =$	— 3	+ 2	+ 3	— 6
	$10^5 \cdot \frac{R_{1m}}{T_1} =$	643	628	606	589
	$\Delta =$	— 11	— 8	— 10	— 5

It has proved that p_m and $\frac{R_{1m}}{T_1}$ may with fairly good approximation be represented by the following empirical expressions

$$\frac{R_{1m}}{T_1} = \frac{k_1 d_1 + k_2}{d_1} \qquad p_m = \frac{k_3 d_1 + k_4}{d_1} \tag{1}$$

where $d_1 = d - B =$ the diameter of the cylinder — the width of the band.

k_1, k_2, k_3, and k_4 are not dependent on d but vary slightly with the temperature, so that we must put

$$
\begin{array}{llll}
k_1 = k_{10}\ (1 + \alpha_1 T_1) & k_{10} = 0.00285 & \alpha_1 = -0.0019 \\
k_2 = k_{20}\ (1 + \alpha_2 T_1) & k_{20} = 0.00404 & \alpha_2 = 0 \\
k_3 = k_{30}\ (1 + \alpha_3 T_1) & k_{30} = 97 & \alpha_3 = +0.0012 \\
k_4 = k_{40}\ (1 + \alpha_4 T_1) & k_{40} = 85 & \alpha_4 = +0.0031
\end{array}
$$

with the numerical values indicated for the quantities k and α.

In the table of p_m and $\frac{R_{1m}}{T_1}$ the difference Δ between the numerical value and the value of the same quantity calculated by means of the above expression is given under each numerical value.

It is obvious that the expressions for p_m and $\frac{R_{1m}}{T_1}$ cannot be employed for very small values of d_1. The case $d_1 << B$ requires special investigation. If, however, we put $d_1 = \infty$, that is to say, if we consider a narrow band in a large cylinder, we get

$$\frac{R_{1m}}{T_1} = k_1 \qquad\qquad p_m = k_3.$$

From this it will appear that k_1 has the same dimensions as $\frac{R_{1m}}{T_1}$ i. e., $\frac{\text{force}}{\text{length}\cdot\text{temperature}}$. The dimensions of k_2, k_3, and k_4 are seen to be respectively $\frac{\text{force}}{\text{temp.}}$, $\frac{\text{force}}{\text{length}^2}$ and $\frac{\text{force}}{\text{length}}$.

For $d_1 = \infty$ p_m and $\frac{R_{1m}}{T_1}$ are assumed to be solely dependent on the width of the band (and the coefficients of accommodation), the kind of gas and the absolute temperature T° (that of the surroundings). The k's must be capable of being expressed by these quantities.

The simplest form in which this can be done is

$$k_1 = c_1 \cdot \frac{\lambda_1}{T_0} \qquad k_3 = c_3 \cdot \frac{\lambda_1}{B}$$

$$k_2 = c_2 \cdot \frac{\lambda_1 \cdot B}{T_0} \qquad k_4 = c_4 \cdot \lambda_1$$

where the c's are numerical constants and $\lambda_1 = p \cdot \lambda =$ the mean free path at a pressure of 1 Bar.

Introducing these values in (1), we obtain

$$\frac{R_{1m}}{T_1} = \frac{c_1 \cdot \frac{\lambda_1}{T_0} \cdot d_1 + c_2 \cdot \frac{\lambda_1 B}{T_0}}{d_1} \qquad p_m = \frac{c_3 \cdot \frac{\lambda_1}{B} \cdot d_1 + c_4 \lambda_1}{d_1}. \tag{2}$$

From this it will appear that both $\frac{R_{1m}}{T_1}$ and p_m should be proportional to λ_1 for all values of B and d_1.

On comparing the ratio $\frac{R_{1m}}{T_1 \lambda_1}$ for various gases, it must, however, be kept in mind that the coefficients of accommodation enter into the numerical constants c, since both c_1 and c_2 must be assumed to be proportional to the difference $a' - a''$ between the coefficients of accommodation for the two sides of the band so that

$$c_1 = \gamma_1 (a' - a''), \quad c_2 = \gamma_2 (a' - a'')$$

and hence

$$\frac{R_{1m}}{T_1 \lambda_1 (a' - a'')} = \frac{\gamma_1 d_1 + \gamma_2 B}{d_1} \cdot \frac{1}{T_0},$$

where the quantity to the right of the sign of equation should be the same for all gases and for large values of d, independent of the width of the band.

This may be tested in the following way. Professor Knudsen has found for the maximum radiometer pressure in hydrogen and helium per 1° temperature difference respectively 0.01342 Dyn/cm² and 0.02625 Dyn/cm², the differences of the coefficients of accommodation being 0.415 and 0.512[1]. The width of the band $B = 0.2484$ cm.

The mean free paths λ_1 in hydrogen and helium are respectively 12.42 and 19.76 cm.

From this we get that the ratio

$$\frac{R_{1m}}{T_1 \lambda_1 (a' - a'')} \quad \text{for hydrogen} = \frac{0.01342 \cdot 0.2484}{12.42 \cdot 0.415} = 0.000647$$

$$\text{for helium} = \frac{0.02625 \cdot 0.2484}{19.76 \cdot 0.512} = 0.000641.$$

With the largest cylinder $d = 4.11$ cm we have for $T_1 = 122°$ $\frac{R_{1m}}{T_1} = 0.00321$ and for $T_1 = 85°$ $\frac{R_{1m}}{T_1} = 0.00342$. Using the value $a' - a'' = 0.415$ we find for the ratio in question respectively

$$\frac{0.00321}{12.42 \cdot 0.415} = 0.000612 \quad \text{and} \quad \frac{0.00342}{12.42 \cdot 0.415} = 0.000652.$$

From the expression for p_m it will be seen that

$$\frac{p_m}{\lambda_1} = \frac{c_3 \cdot \frac{d_1}{B} + c_4}{d_1}.$$

For p_m in hydrogen and helium Professor Knudsen found $p_m = 134$ and 191 Bars.

The ratio $\frac{p_m}{\lambda_1}$ is then for hydrogen $= \frac{134}{12.42} = 10.8$

and for helium $= \frac{191}{19.76} = 9.7.$

The difference (11 %) between the two ratios is somewhat larger than might be expected even though the determination of p_m is essentially more uncertain than the determination of the radiometer force.

[1] L. c. p. 56.

With regard to the influence of the width of the band on p_m and $R_{1,m}$ the expressions (2) give:

When d is large, $\frac{R_{1m}}{T_1}$ is but slightly dependent on B, whereas p_m is approximately inverse to B.

This relationship has not as yet been experimentally investigated since, as will easily be understood, the results of experiments with radiation radiometers cannot be employed.

Each individual radiometer curve may with good approximation be represented by an interpolation formula

$$\frac{R_1}{T_1} = \frac{ap}{1 + bp + b^2 p^2 + c^3 p^3} \tag{3}$$

where a, b, and c are constants for each curve.

Differentiating with respect to p we find

$$\frac{d\frac{R_1}{T_1}}{dp} = 0 \quad \text{for} \quad b^2 p_m{}^2 + 2c^3 p_m{}^3 = 1.$$

If the member $c^3 p^3$ which, since c^3 is very small, will only influence the very highest pressures, is omitted from the denominator, the formula will give a symmetrical radiometer curve in a log — linear representation.

The empirical curves all deviate appreciably from symmetry, the radiometer force at the highest pressures (1000 10000 Bars) being less than it should be if the curve were symmetrical.

This has necessitated the inclusion of the member $c^3 p^3$ in the denominator so as to obtain a tolerably good presentation.

The numerical value of the constant c is for all curves approximately $\frac{1}{3}\cdot b$, though the ratio between b and c seems to be somewhat dependent on the diameter d of the cylinder, $\frac{c}{b}$ increasing somewhat when d decreases.

If, in order to facilitate calculation, we put $c = \frac{1}{3}\cdot b$, we find

$$b^2 p_m{}^2 + 2\cdot\frac{1}{27}b^3 p_m{}^3 = 1$$

and from this $b\cdot p_m = 0.9660$.

If we had put $c = 0$, we should have obtained $b\cdot p_m = 1$.

Introducing the value $b = \frac{0.9660}{p_m}$ in expression (3), we obtain

$$a = 2.9326\cdot\frac{R_{1m}}{T_1\cdot p_m},$$

when $c = \frac{1}{3}\cdot b$, $b = \frac{1}{p_m}$ giving $a = 3.\ \frac{\frac{R_{1m}}{T_1}}{p_m}$ when $c = 0$.

Using the values of b and a corresponding to $c = \frac{1}{3}\cdot b$ and introducing expressions (1) and (2) for $\frac{R_{1m}}{T_1}$ and p_m, we obtain

$$\left.\begin{aligned} a &= 2.9326\cdot\frac{c_1 d_1 + c_2 B}{c_3\cdot\frac{d_1}{B} + c_4}\cdot\frac{1}{T_0} = 2.9326\cdot\frac{k_1 d_1 + k_2}{k_3 d_1 + k_4} \\ b &= 0.9660\cdot\frac{d_1}{c_3\cdot\frac{\lambda_1}{B}\cdot d_1 + c_4\lambda_1} = 0.9660\cdot\frac{d_1}{k_3 d_1 + k_4} \\ \frac{R_1}{T_1} &= \frac{ap}{1 + bp + b^2p^2 + \left(\frac{1}{3}bp\right)^3}, \end{aligned}\right\} \quad (4)$$

with the previously given numerical values of the quantities k and α.

In fig. 3 the calculated curve of $\frac{R_1}{T_1}$ is dotted. The agreement between the calculated and experimental curves is about the same for the other 15 radiometer curves.

For low pressures expressions (4) give

$$\frac{R_1}{T_1} = \frac{2.9326\,(c_1 d_1 + c_2 B)}{c_3 \frac{d_1}{B} + c_4} \cdot \frac{p}{T_0} \tag{5}$$

from which it is seen that when $B << d$, the radiometer force is proportional to the width of the band (surface effect) and independent of d_1. As mentioned above, the constants c_1 and c_2 are proportional to the difference between the coefficients of accommodation $c_1 = \gamma_1\,(a' - a'')$, $c_2 = \gamma_2\,(a' - a'')$.

For large values of d_1 the expression is in perfect agreement with Professor KNUDSEN's formula for the radiometer pressure at low pressures, which is

$$\frac{p' - p''}{T_1} = \frac{1}{4} p \cdot \frac{a' - a''}{T_0}$$

(T_1 = temperature difference) whilst the above formula gives

$$\frac{R_1}{T_1} = 2.9326 \cdot B \cdot p \cdot \frac{a' - a''}{T_0} \cdot \frac{\gamma_1}{c_3}$$

but

$$\frac{R_1}{B} = p' - p'', \quad \text{and} \quad \frac{\gamma_1}{c_3}$$

is dimensionsless.

When B and d_1 are of the same order of magnitude, expression (5) does not show strict proportionality with B nor strict independence of d_1; this is in good agreement with the measurements at low pressures, a slight increase of $\frac{R_1}{T_1}$ having been found with decreasing d_1.

Measurements at low Pressures

were only made with the temperature difference $T_1 = \text{c. } 122°$. R_1 being very nearly proportional to the pressure in this field, we only give the approximate value of p and the quotient $\frac{R_1}{p \cdot T_1}$.

Table of $10^6 \cdot \frac{R_1}{p \cdot T_1}$.

p	$d = 4.11$ cm	$d = 3.21$ cm	$d = 2.24$ cm	$d = 1.27$ cm
2	61	55	69	84
4	59	56	67	79
6	57	56	67	77
8	59	57	66	76
10	59	59	66	75
12	57	58	67	75
14	58	58	66	76
16	57	58	65	73
18	57	58	65	74
20	55	58	65	73
22	..	..	65	72
24	..	..	65	71

From (4) we find for large values of p $\frac{R_1}{T_1} = 27 \cdot \frac{a}{b^3 p^2}$. For large values of the diameter d_1 $a = \text{constant} \cdot \frac{B}{T_0}$ and $b = \text{constant} \cdot \frac{B}{\lambda_1}$, hence $\frac{R_1}{T_1}$ becomes $= \text{constant} \cdot \frac{\lambda_1^3}{B^2 p^2 T_0}$. That R_1 is here found to be inverse to B^2 can hardly be correct. One would have expected independence of B or, at most, B in the denominator.

Here we would remark, however, that by putting the ratio $\frac{c}{b} = \text{constant} = \frac{1}{3}$, which was quite satisfactory for the numerical application of the interpolation formula, we have at the beginning introduced the presupposition that the radiometer force should, at the highest pressures, be solely dependent on the same factors which determine R_m and p_m. This presupposition, however, is hardly correct since it must be assumed that the thickness τ of the band, which at high pressures will be of the same order of magnitude as the mean free path, must also enter into the expression for the radiometer force.

c^3 is to be of the same dimension as b^3 or as $\left(\frac{B}{\lambda_1}\right)^3$. This may be attained by putting $c^3 = \text{constant} \cdot \frac{\tau}{\lambda_1} \cdot b^2$ which for large values of d gives $c^3 = \text{constant} \cdot \frac{\tau \cdot B^2}{\lambda_1^{\,3}}$.

From this we obtain $\frac{R_1}{T_1} = \text{the numerical constant} \cdot \frac{\lambda_1^{\,3}}{B \tau p^2 T_0}$.

The experiments here under consideration do not allow of any proof of the correctness of this expression.

P. S. Epstein has calculated the radiometer force acting at high gas pressures on a circular disc, between the two sides of which there is a temperature difference of ΔT. The force is due to the thermal sliding of the gas round the edge of the disc from the cold to the warm side. He finds[1]

$$K = \frac{8}{3} \cdot \frac{p \cdot \lambda^2}{T_0} \cdot \Delta T = \frac{8}{3} \cdot \frac{\lambda_1^{\,2}}{p} \cdot \frac{\Delta T}{T_0}.$$

Hence the force for constant ΔT is in inverse proportion to p and in direct proportion to $\lambda_1^{\,2}$.

Epstein's coefficient of accommodation being 1, his ΔT denotes the difference between the gas temperatures on the two sides of the disc.

When the platinum band in our experiment is heated to T_1° above the temperature of the cylinder, there occurs on either side of the band a rise in temperature of $\Delta T'$ and $\Delta T''$ which at high pressures is proportional to the mean free path and otherwise depends on the coefficient

[1] Zeitschrift f. Physik 54 p. 556, 1929.

of accommodation and the temperature gradient in the gas. For the rise in temperature with a plane surface Professor Knudsen gives the following formula[1]

$$\Delta T' = \frac{2-a'}{2a'} \cdot k \cdot \lambda \cdot \frac{dT}{dx},$$

where k is a constant.

If this is assumed to hold good for our platinum band, the temperature difference becomes

$$\Delta T = \Delta T'' - \Delta T' = \frac{a'-a''}{a'a''} \cdot k \cdot \lambda \cdot \frac{dT}{dx}$$

$$= \text{const. } (a'-a'') \frac{\lambda_1}{p} \cdot \frac{dT}{dx}$$

where $\frac{dT}{dx}$ = the temperature gradient at the surface of the platinum band which is here assumed to be the same on both sides.

$\frac{dT}{dx}$ is proportional to T_1 and dependent on the dimensions of the apparatus.

On the assumption that the pressure dependence calculated by Epstein also holds good for the radiometer force on the platinum band, introducing the expression for ΔT, we find

$$R_1 = \text{const. } (a'-a'') \cdot \frac{\lambda_1^3}{p^2} \cdot \frac{dT}{dx}$$

which is in good agreement with the expressions given above for $\frac{R_1}{T_1}$ at high pressures with respect to the factor $\frac{\lambda_1^3}{p^2}$.

That the deviation from symmetri of the radiometer curve at high pressures is due to horizontal currents is further supported by the fact that the conduction of heat

[1] Martin Knudsen D. K. D. V. S. Forh. 1911 No. 2, p. 199.

within the range of pressures where the deviation from symmetry is great (1000—10000 Bars) is no longer proportional to T_1, but increases more rapidly than T_1, which can only mean that, in addition to the ordinary conduction, a conveyance of heat by currents takes place. SOPHUS WEBER has investigated the effect of convention caused by changes in density[1] and pointed out that it is small in an apparatus with a vertical filament or band and notably small at low pressures. WEBER's measurements were made at pressures between 1 atmosphere and c. 0.2 mm Hg = c. 2500 Bars, his lowest pressure thus corresponding to our highest. Hence it is evident that the conveyance of heat by convection here observed is in the main due to horizontal currents.

We may say, therefore, that the deviation of the radiometer curve from symmetry is the greater, the greater the positive temperature coefficient to $\frac{q}{T_1}$ at high pressures.

The following table shows the value of the temperature coefficient α, with $\frac{q}{T_1} = \left(\frac{q}{T_1}\right)_0 \cdot (1 + \alpha T_1)$; we give the value of $\alpha \cdot 10^5$ for $p = 2000$ and 5000 Bars

d	$p = 2000$	$p = 5000$
4.11	65	93
3.21	47	83
2.24	47	81
1.27	55	75

At low pressures and about p_m α is very approximately $= 0$.

[1] Undersøgelser over Luftarternes Varmeledningsevne, Copenhagen 1916. See p. 48.

If it is desired to ascertain the course of R_1 for a constant heat supply per cm^2 of the band and per sec. in order to compare the results with the experimental results for radiation radiometers, the conversion from R_1/(temperature difference) to R_1/(power) may easily be done by means of the values of heat loss by radiation and conduction known from the measurements.

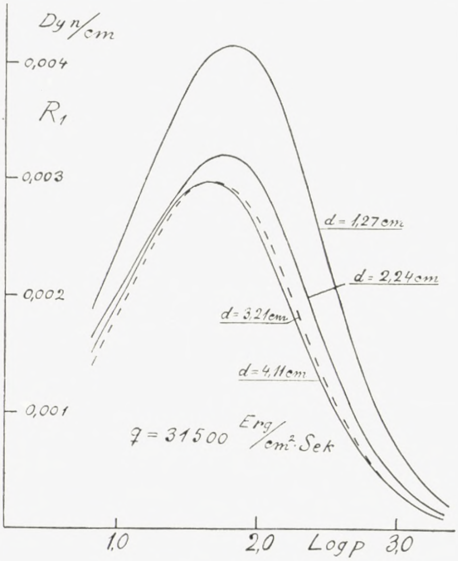

Fig. 4.

For the details the reader is referred to Professor Knudsen's aforementioned paper[1].

Fig. 4 shows the main result of this calculation. The ordinates of the 4 curves are R_1, corresponding to an arbitrarily selected but constant supply of energy per cm^2. per sec.

As will be seen, the pressure p_m at which R_1 is maximum for q = constant is less than (c. $^1/_3$ of) p_m for T_1 = constant, and it will be noted that though $R_{1,m}$ likewise increases with decreasing d for q = constant it is not nearly so much as for T_1 = constant.

In further illustration hereof we give the relative values of R_m for the different cylinders

	d = 4.11 cm	3.21 cm	2.24 cm	1.27 cm
R_{1m} for T_1 = const.	= 1	1.07	1.27	1.88
R_{1m} for q = const.	= 1	1.00	1.08	1.40

$R_{1,m}$ for the largest cylinder being in both cases put = 1.

[1] L. c. p. 63.

That the changes would tend in this direction could be known at the outset.

For, if T_1 is kept constant and the wide cylinder ($d = 4.11$ cm) is exchanged for the narrow one ($d = 1.27$), we obtain for a certain pressure p, say a times as large a value of R_1, where $a \geqq 1$, but varies with p.

If, on the other hand, q is kept constant and we make the same exchange, we obtain for the same pressure p, say b times as large a value of T_1 where $b \leqq 1$, but varies with p.

Hence for R_1 which is very nearly proportional to T_1, we obtain by the latter exchange an $a \cdot b$ larger value; but $a \cdot b \leqq a$.

This furnishes a qualitative explanation of PRINGSHEIMS and NICHOLS's observation that in the construction of a sensitive radiometer the vane system should not be placed as close as possible to the wall of the container; on the contrary, the greatest sensitivity, i. e. the greatest radiometer force, for a given radiation is obtained by placing it at a certain small distance from the wall.

On comparing the above-mentioned results with experimental data for radiation radiometers it should always be kept in mind that in the radiation apparatus there occurs an effect due to the heating of the container by absorption of the radiation, and that this must be taken into account.

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